Utility Maximization with Perfect Complements 2

Consider a consumer who derives utility from consuming two goods, x_1 and x_2 The consumer's utility function is given by

$$U(x_1, x_2) = \min\{2x_1, 3x_2\}$$

where the constants are $a = 2$ and $b = 3$ The consumer faces a budget constraint

$$4x_1 + 5x_2 = 100$$

where $p_1 = 4$ and $p_2 = 5$ are the prices of goods 1 and 2, respectively, and $M = 100$ is the consumer's income Determine the optimal consumption bundle (x_1^*, x_2^*) and the optimal utility U^*

Solution

For perfect complements, the consumer will consume the two goods in fixed proportions determined by the constants in the utility function The optimal consumption ratio is

$$\frac{x_1}{x_2} = \frac{b}{a} = \frac{3}{2}$$

Thus, we have

$$x_1 = \frac{3}{2}x_2$$

Substitute this expression into the budget constraint

$$4\left(\frac{3}{2}x_2\right) + 5x_2 = 100$$

Simplify the equation

$$6x_2 + 5x_2 = 100$$

$$11x_2 = 100$$

Solving for x_2, we find

$$x_2^* = \frac{100}{11}$$

Next, substitute x_2^* back to determine x_1^*

$$x_1^* = \frac{3}{2} \cdot \frac{100}{11} = \frac{150}{11}$$

Optimal Utility

The optimal utility is found by substituting the optimal bundle into the utility function

$$U^* = \min\{2x_1^*, 3x_2^*\} = \min\left\{2 \cdot \frac{150}{11}, 3 \cdot \frac{100}{11}\right\}$$

Compute both expressions

$$2x_1^* = 2 \cdot \frac{150}{11} = \frac{300}{11} \quad 3x_2^* = 3 \cdot \frac{100}{11} = \frac{300}{11}$$

Since both expressions are equal, the optimal utility is

$$U^* = \frac{300}{11}$$